Filtering and aliasing effects produced by time-discretisation in the simulation of Extensive Air Shower radio signals

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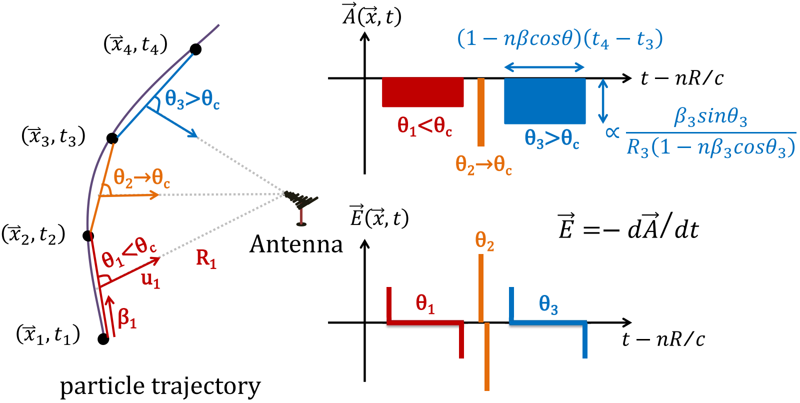
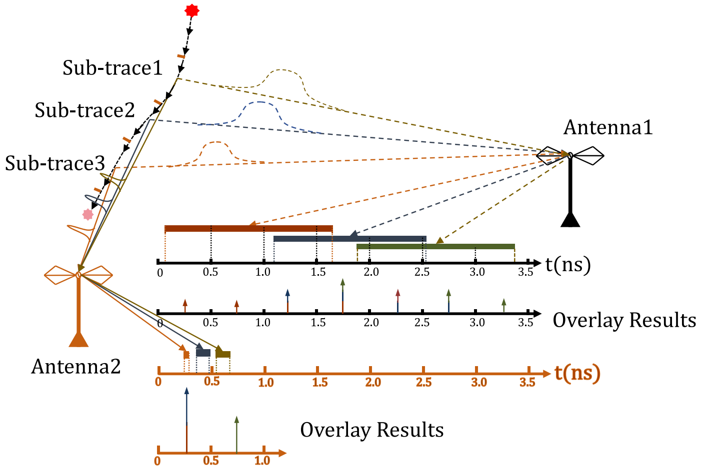
Introduction: Modelling EM emission in simulations

- Particle trajectories subdivided in straight tracks
- Model: Infinite acceleration at the ends of the track
- Associated vector potential: Boxcar shape



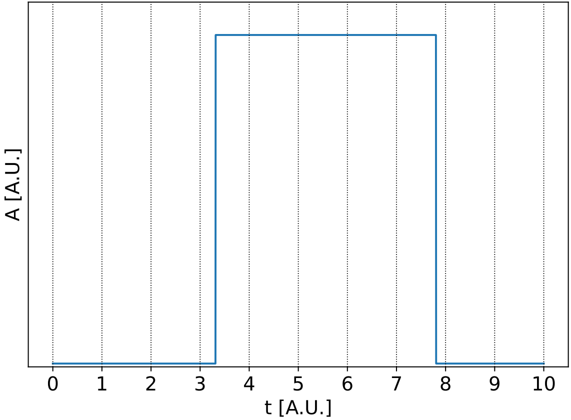
Time binning

- Any implementation needs to sample the vector potential or electric field at a finite set of points
- Final pulse built by adding the contributions inside each time bin



Sampling algorithm in ZHS

- In ZHS, the sampled value is the average of **A** inside each time bin.



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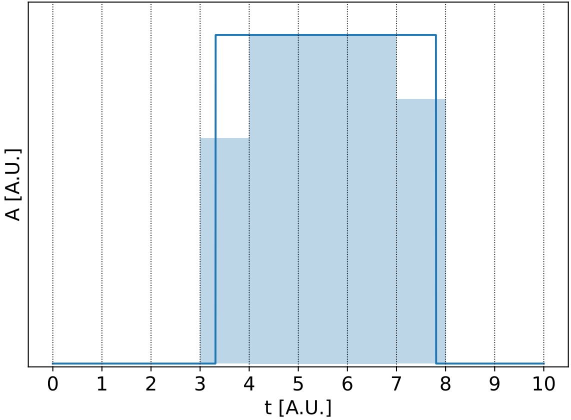

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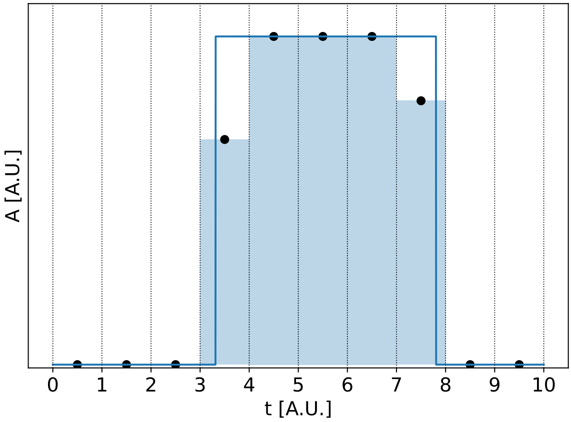

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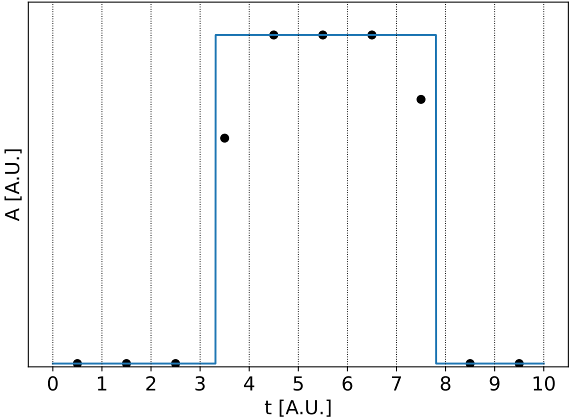

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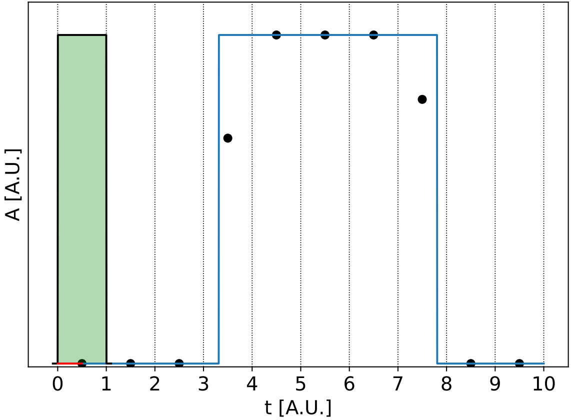

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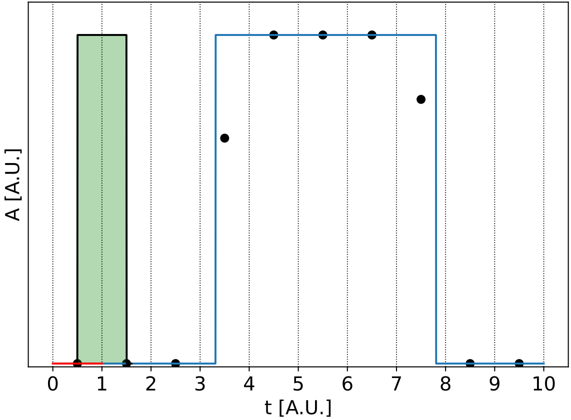

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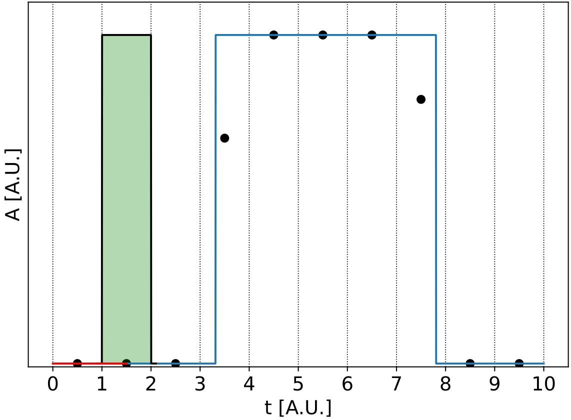

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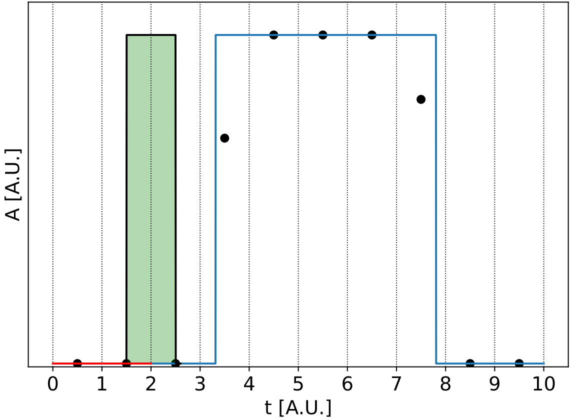

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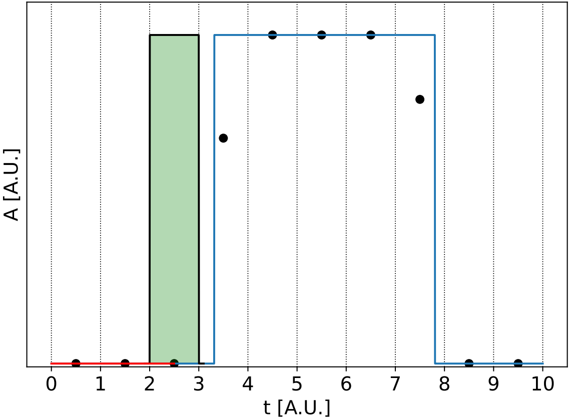

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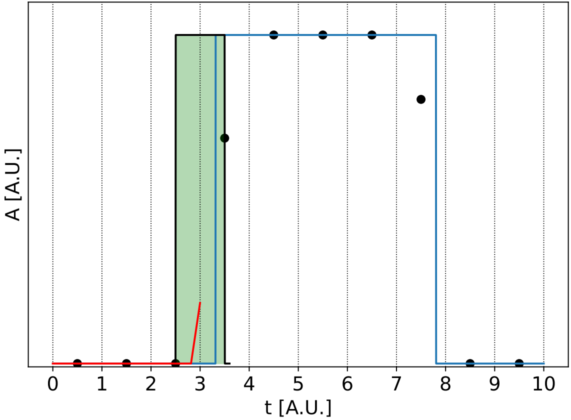

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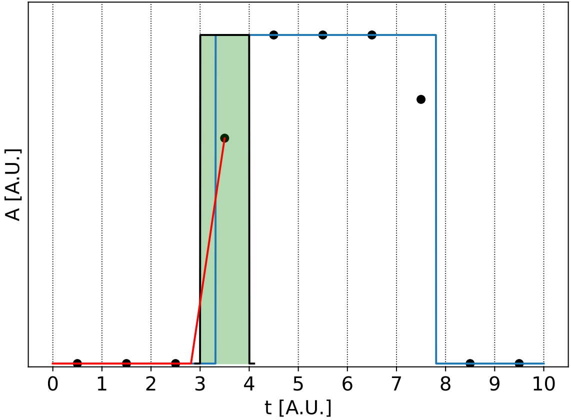

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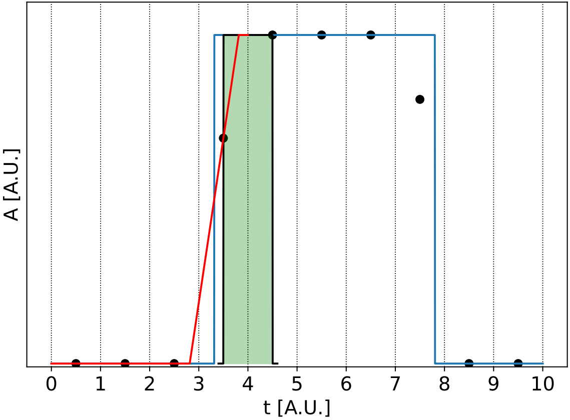

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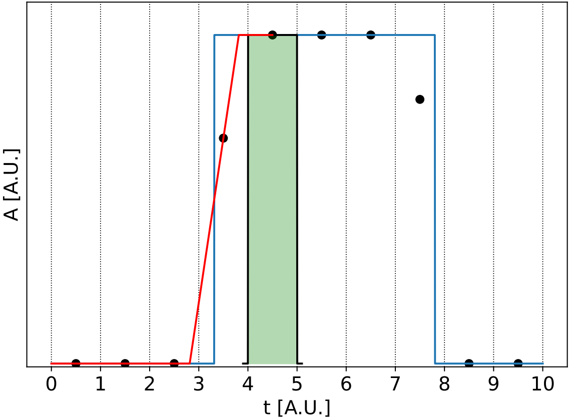

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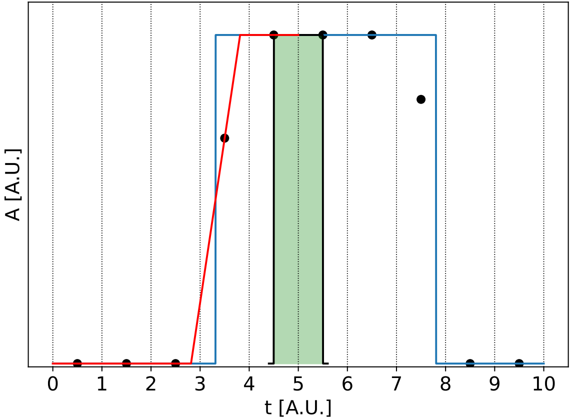

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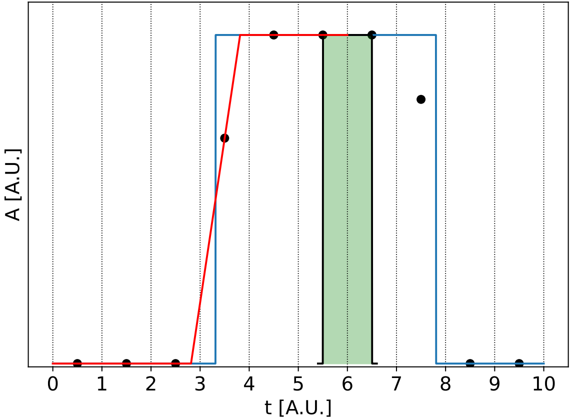

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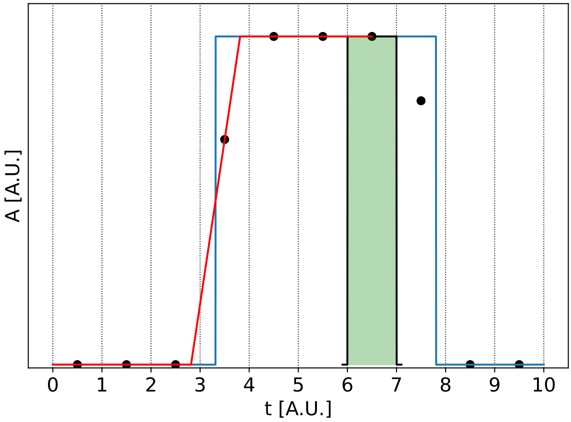

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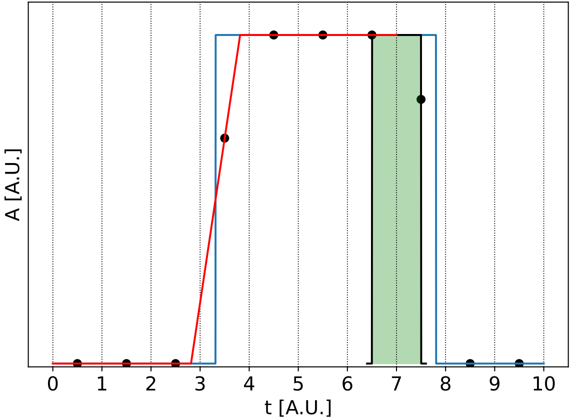

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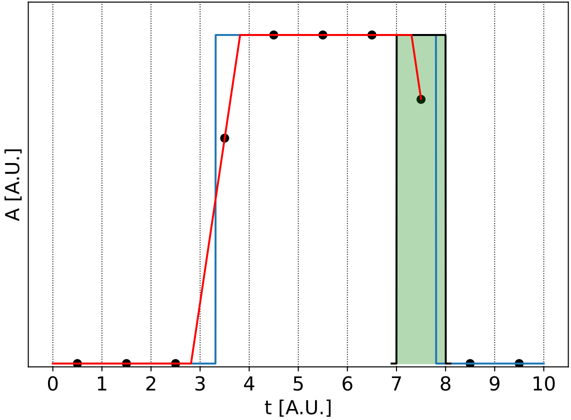

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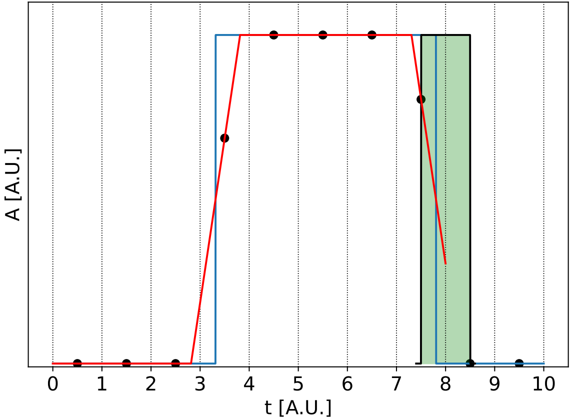

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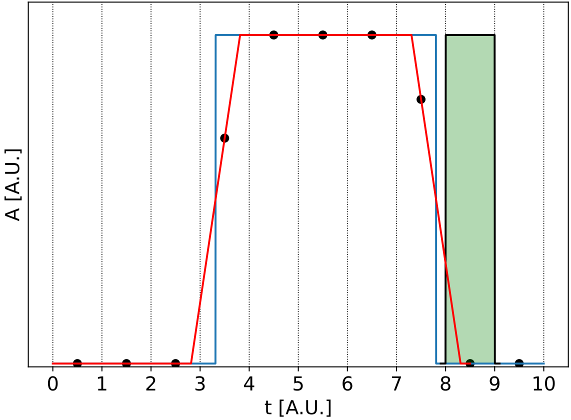

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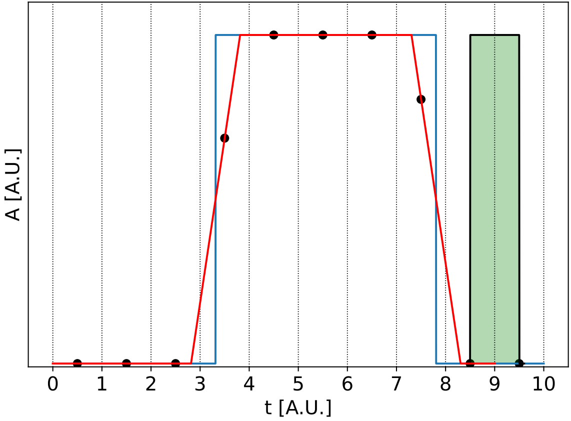

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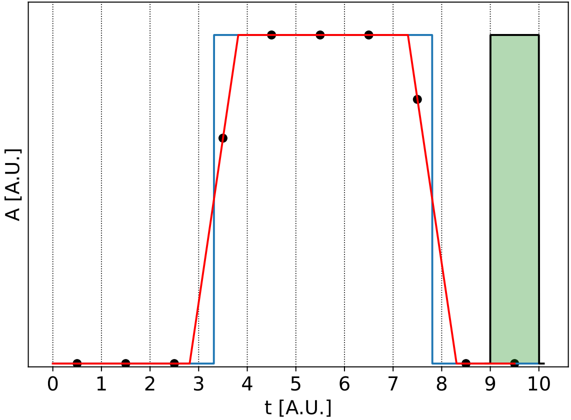
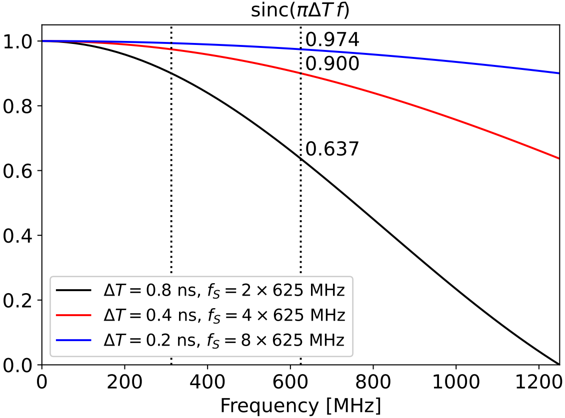


- We are sampling a convolution with a *unit* boxcar function (moving average)

Moving average & sinc filters

- Convolution in time domain $\leftrightarrow$ Product in frequency domain
- Normalized boxcar of width $\Delta T \leftrightarrow \mathcal{F} \leftrightarrow \mathrm{sinc}\,(\pi \Delta T\, f)$
- We are secretly applying a sinc filter



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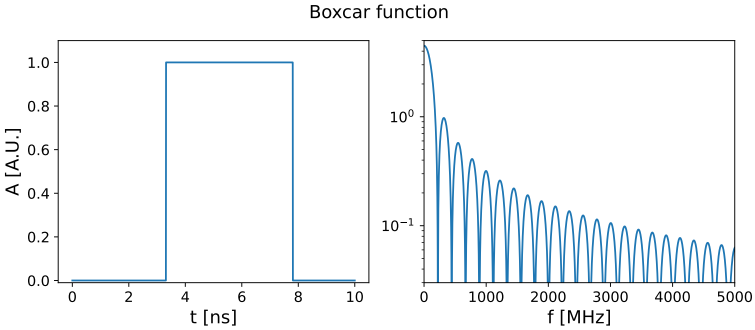
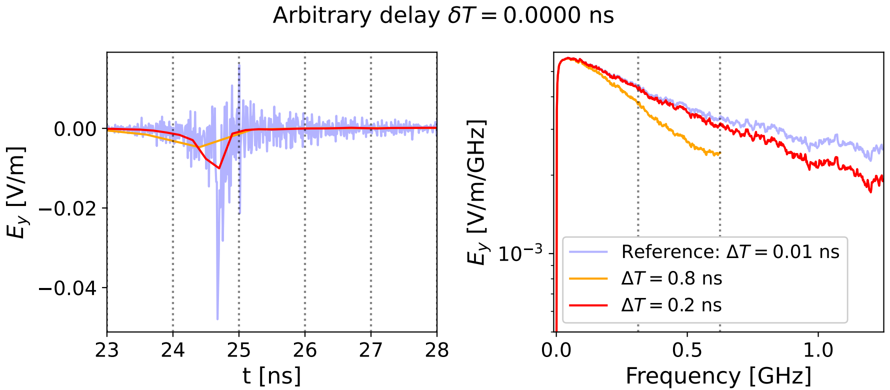
- Only if the signal has no frequency content above f
- Otherwise, aliasing effects will appear
- But we are sampling signals with infinite frequency content!



Spectral distortion

- Example: Antenna at the Cherenkov angle in a $\theta = 67^\circ$ shower.
- Pulse built using the ZHS algorithm
- Apply arbitrary shift relative to first time bin



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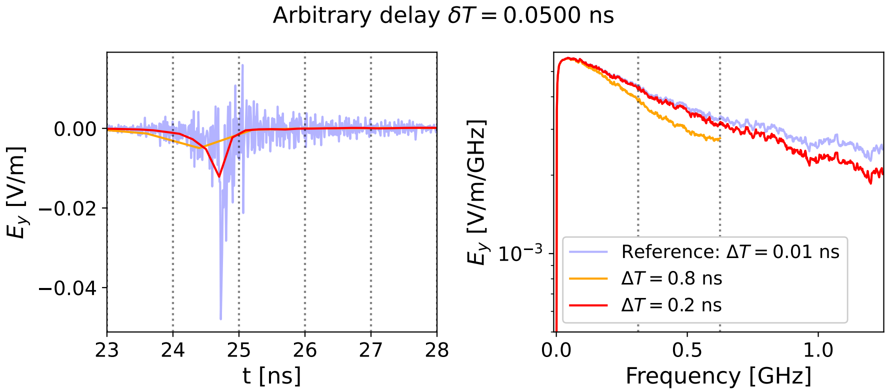

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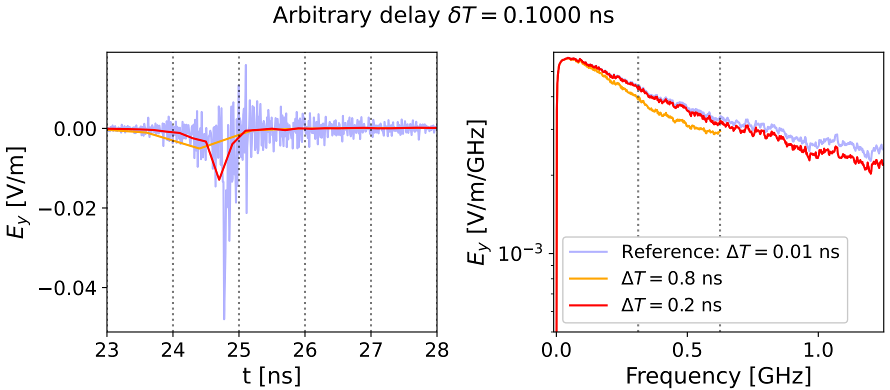

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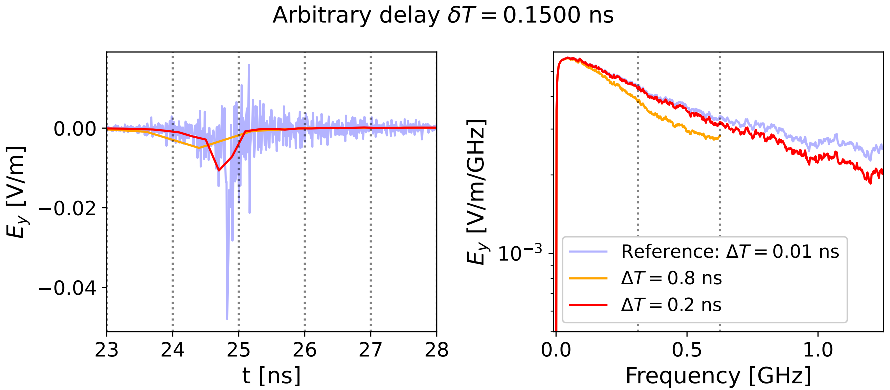

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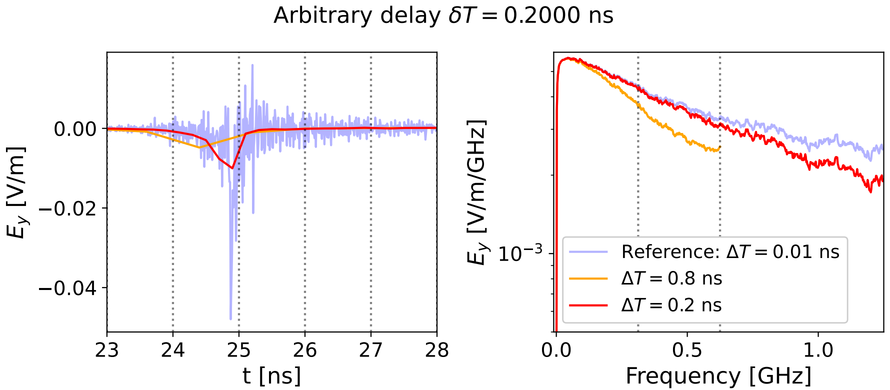

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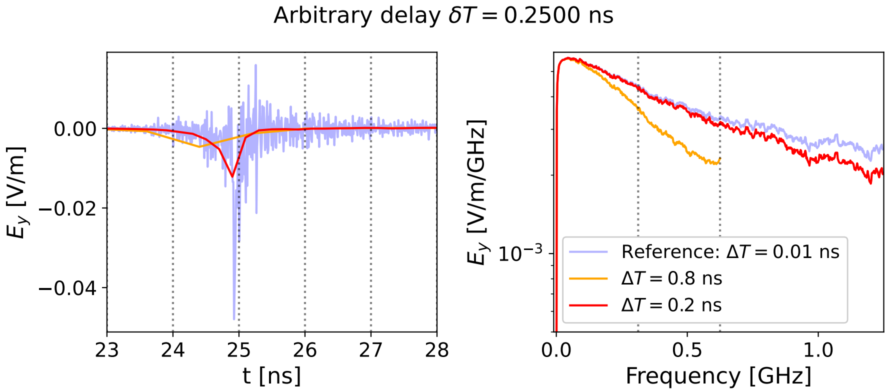

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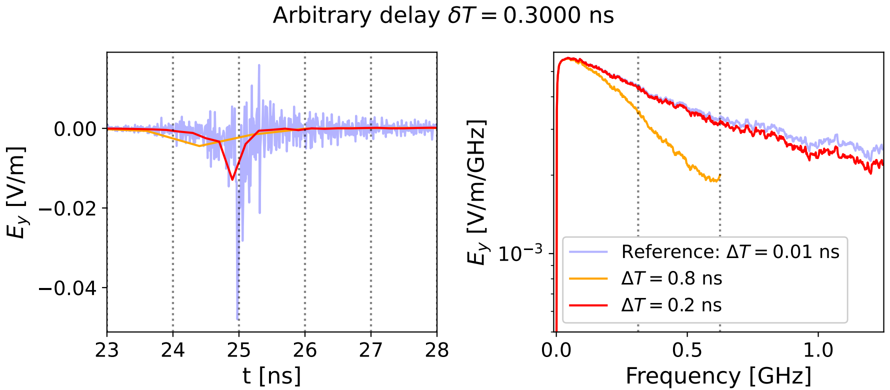

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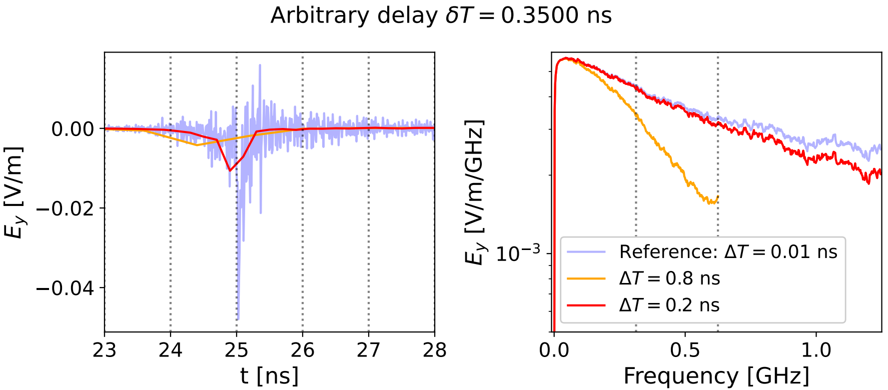

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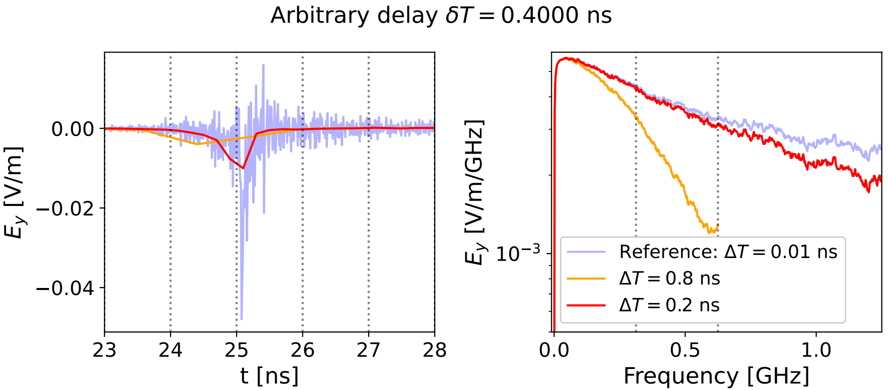

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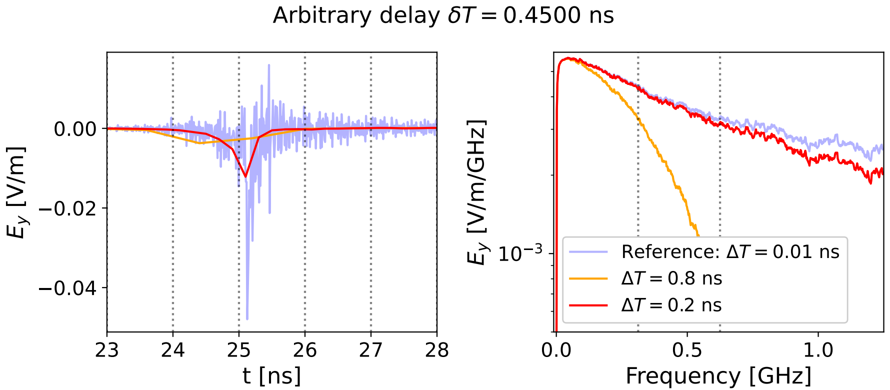

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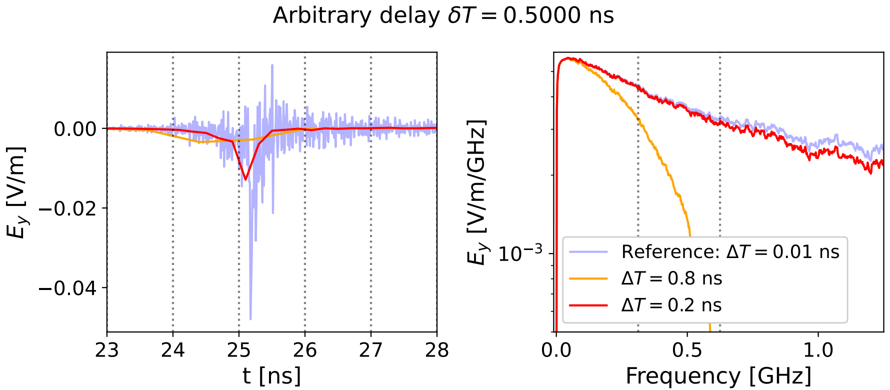

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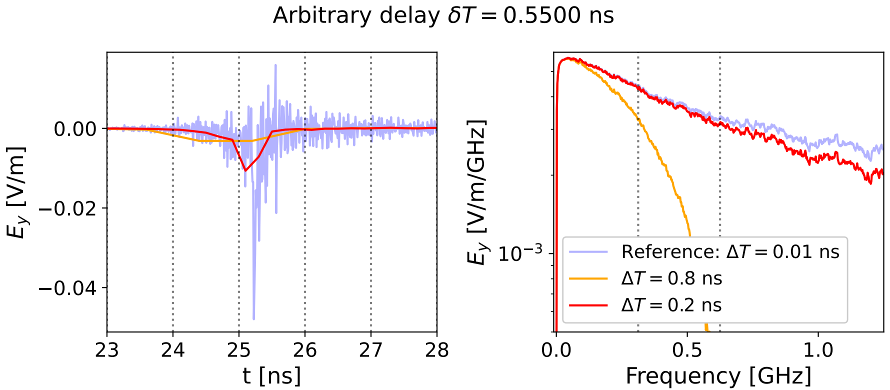

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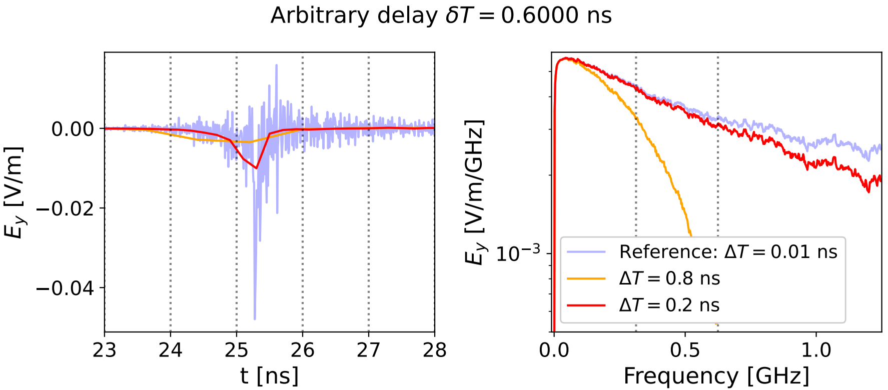

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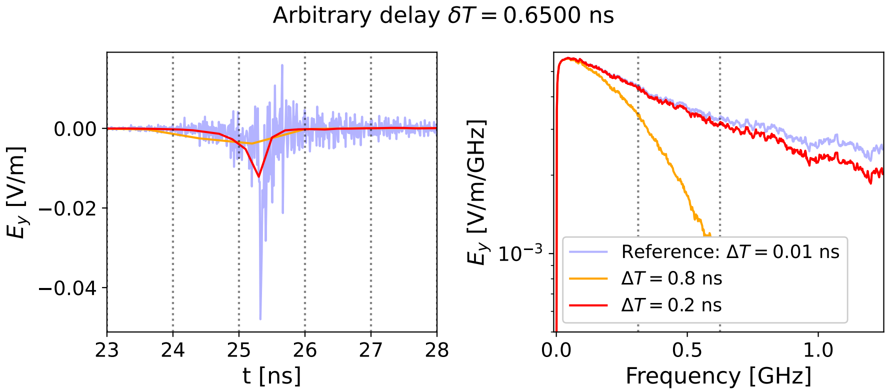

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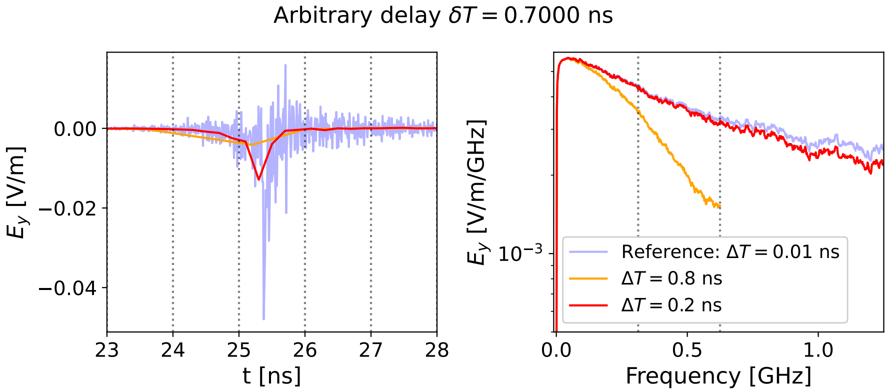

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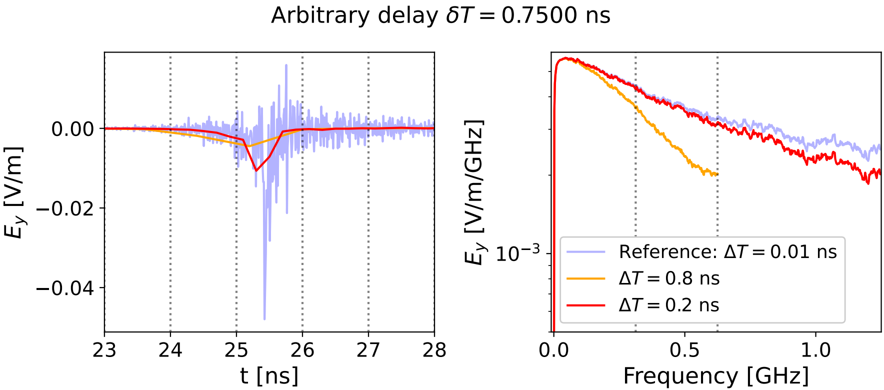
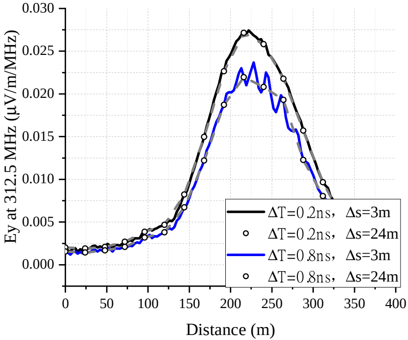


LDF at a constant frequency

- Amplitude differences: Moving average filters
- Ripples: Phase differences between close antennas (usually unnoticed)



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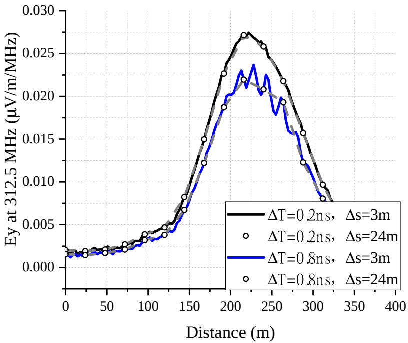
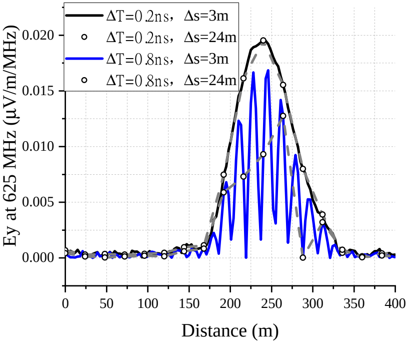
- Important effect close to the Nyquist frequency





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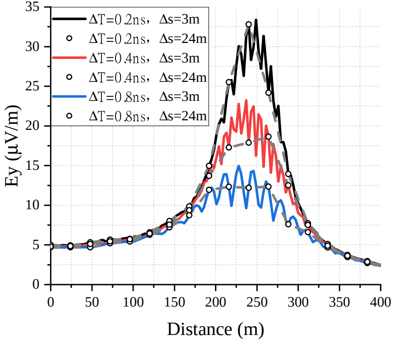
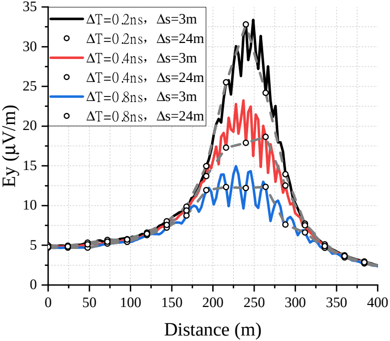
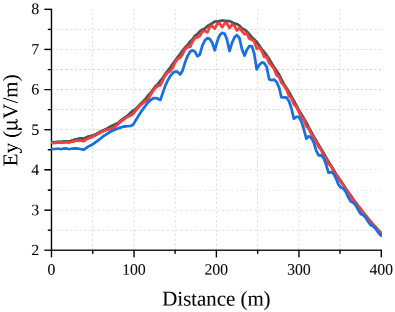


LDF: Maximum amplitude of electric field

- Amplitude differences: Moving average filters
- Ripples: Phase differences between close antennas (usually unnoticed)
- Filtering the signal after simulation is not a solution for aliasing



< 250 MHz →



Conclusions

- Current sampling algorithms are equivalent to moving-average filters
- Emission models are based in infinite-bandwidth vector potentials or electric fields
- Intrinsic distortions in the spectra appear due to aliasing
- Increased sampling rates and post-simulation processing are useful but do not solve the underlying problem.

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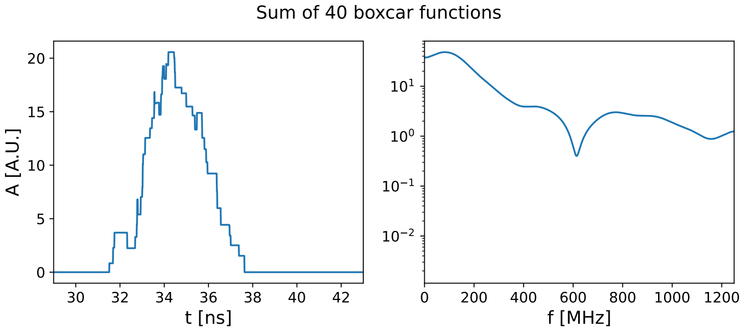
Some ideas

- *Adaptative* sampling rates depending on the antenna
- Build pulses from finite-bandwidth contributions

Backup: Example of aliasing in a simple case

- 40 boxcar functions with gaussian distributed amplitude, width and starting point.



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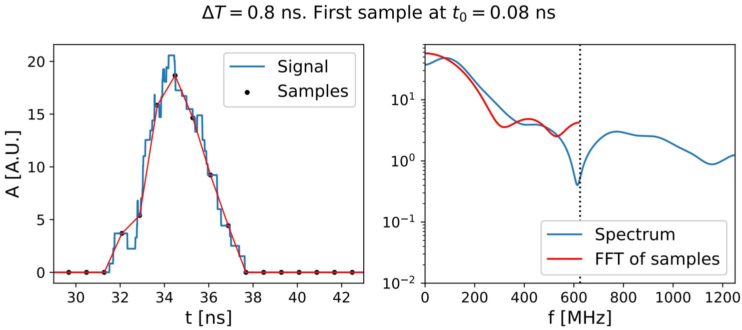

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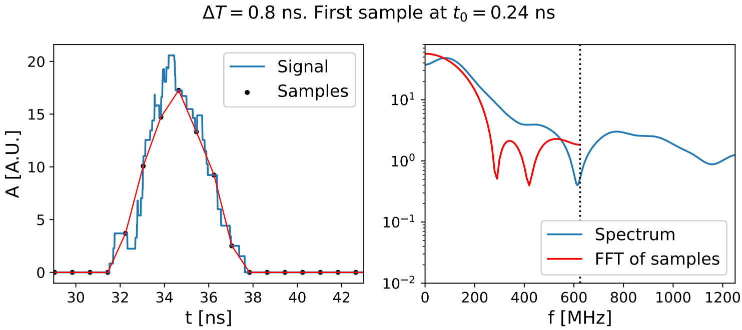

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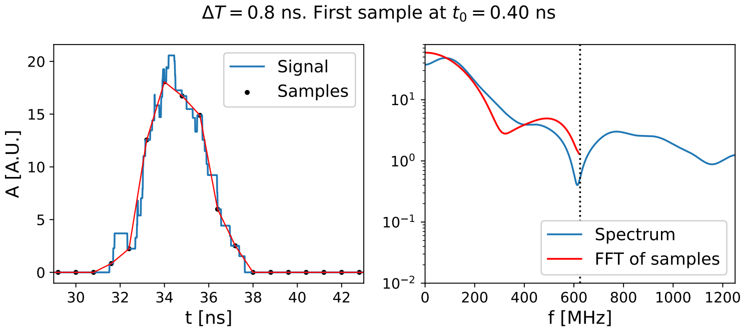

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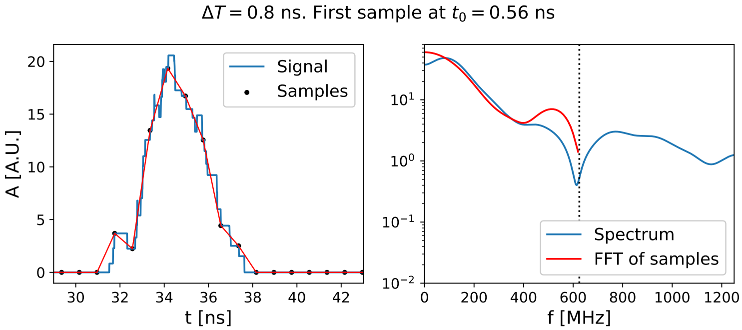

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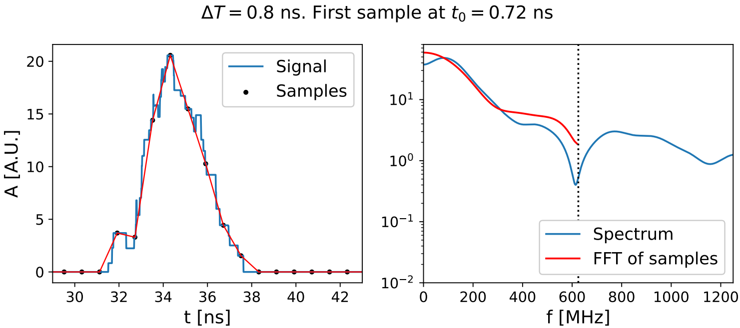

Backup: Computing resources

- Increasing the sampling rate comes at a cost in CPU time and disk usage
- Close to the Cherenkov angle many tracks will always be narrower than the time bin